PHYS 331 – Assignment #3

Due Monday, Nov. 20 at 08:00

1. Find and list 4 references that can be used for your Experiment #2. None can be websites and only one can be a textbook. None can be the material that was provided to you on the Canvas website. In a few short sentences, describe why each reference will be a valuable resource for your project.

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In the seminar we discussed Fourier transforms of non-periodic signals $f(t)$. In particular we considered pulses that can have any arbitrary shape but go to zero for $t \to \pm\infty$. The Fourier transform of signal $f(t)$ was denoted $\hat{f}(\omega)$ and it gives the frequency profile (or content) of the signal. In general we saw that short pulses contain a wide range of frequencies while wide pulses have a narrow frequency spectrum. The Fourier transform of $f(t)$ is given by:

$$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t}\, dt$$

2. First, let's examine some general properties of Fourier transforms. Prove that the following properties are true:

i. If $g(t) = f(t+b)$, then $\hat{g}(\omega) = e^{j\omega b}\hat{f}(\omega)$

ii. If $g(t) = f(at)$, then $\hat{g}(\omega) = \frac{1}{a}\hat{f}\left(\frac{\omega}{a}\right)$

iii. $\hat{\dot{f}}(\omega) \equiv \int_{-\infty}^{\infty} \dot{f}(t) e^{-j\omega t}\, dt = j\omega \hat{f}(\omega)$ where $\dot{f}(t) = df/dt$.

3. Find the Fourier transform $\hat{f}(\omega)$ of the following pulse:

$$f(t) = \begin{cases} 1 - \frac{|t|}{a}, & -a \leq t \leq a \\ 0, & \text{otherwise} \end{cases}$$

On a single graph, plot $\hat{f}(\omega)$ as a function of ω for $a = 0.5$, 1, and 2.

4. Find the Fourier transform $\hat{g}(\omega)$ of a Gaussian pulse:

$$g(t) = \frac{1}{\sigma_t\sqrt{2\pi}} e^{-t^2/(2\sigma_t^2)}$$

(a) Specifically, show that $\hat{g}(\omega)$ is also Gaussian (albeit, not normalized).

(b) What is the relationship between the width σ_ω of the $\hat{g}(\omega)$ distribution and the width σ_t of the $g(t)$ distribution?

(c) Use de Broglie's expression for energy to show that this analysis implies:

$$\sigma_E \sigma_t = \hbar$$

which is reminiscent of the Heisenberg Uncertainty Principle (HUP).